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# A half-space problem in the theory of generalized thermoelastic diffusion

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**Abstract**

In this work we consider the problem of a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. The bounding surface of the half-space is taken to be traction free and subjected to a time dependent thermal shock. The chemical potential is also assumed to be a known function of time on the bounding plane. Laplace transform techniques are used. The solution is obtained in the Laplace transform domain by using a direct approach. The solution of the problem in the physical domain is obtained numerically using a numerical method for the inversion of the Laplace transform based on Fourier expansion techniques.

The temperature, displacement, stress and concentration as well as the chemical potential are obtained. Numerical computations are carried out and represented graphically.


*Keywords:* Thermoelastic diffusion; Generalized thermoelasticity; Thermal shock

## 1. Introduction

Biot (1956) developed the coupled theory of thermoelasticity to deal with a defect of the uncoupled theory that mechanical causes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves.

Lord and Shulman (1967) introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended by Sherief (1980) and Dhaliwal and Sherief (1980) to include the anisotropic case. In this theory, a modified law of heat conduction including

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both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and coupled theories of thermoelasticity. For this theory, Ignaczak (1982) studied uniqueness of solution; Sherief (1987) proved uniqueness and stability. Anwar and Sherief (1988a) and Sherief (1993) developed the state space approach to this theory. Anwar and Sherief (1988b) completed the integral equation formulation. Sherief and Hamza (1994) and Sherief and Hamza (1996) solved some two-dimensional problems and studied wave propagation. Sherief and El-Maghraby (2003) solved a problem for an internal penny shaped crack.

Diffusion can be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in MOS transistors and dope poly-silicon gates in MOS transistors. In most of these applications, the concentration is calculated using what is known as Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction.

Nowacki (1974a,b,c,d) developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Recently, Sherief et al. (2004) developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves.

## 2. Formulation of the problem

We consider the problem of an isotropic thermoelastic half-space ($x \geqslant 0$) with a permeating substance (such as a gas) in contact with the upper plane of the half-space ($x = 0$). The $x$-axis is taken perpendicular to the upper plane pointing inwards. This upper plane of the half-space is taken to be traction free and subjected to a time dependent thermal shock. The chemical potential is also assumed to be a known function of time on the upper plane. All considered functions are assumed to be bounded and vanish as $x \to \infty$.

The equation of motion in the absence of body forces is given by Sherief et al. (2004)

$$\rho \ddot{u}_i = \mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i}, \tag{1}$$

where $u_i$ are the components of the displacement vector, $T$ is the absolute temperature, $C$ is the concentration of the diffusive material in the elastic body, $\lambda$, $\mu$ are Lamé's constants, $\rho$ is the density $\beta_1$ and $\beta_2$ are material constants given by

$$\beta_1 = (3\lambda + 2\mu)\alpha_t \text{ and } \beta_2 = (3\lambda + 2\mu)\alpha_c,$$

$\alpha_t$ is the coefficient of linear thermal expansion and $\alpha_c$ is the coefficient of linear diffusion expansion.

The energy equation has the form Sherief et al. (2004)

$$kT_{,ii} = \rho c_E(\dot{T} + \tau_0 \ddot{T}) + \beta_1 T_0(\dot{e}_{kk} + \tau_0 \ddot{e}_{kk}) + aT_0(\dot{C} + \tau_0 \ddot{C}), \tag{2}$$

where $k$ is the thermal conductivity, $c_E$ is the specific heat at constant strain, $\tau_0$ is the thermal relaxation time, '$a$' is a measure of thermodiffusion effect and $T_0$ is a reference temperature assumed to obey the inequality $|(T - T_0)/T_0| \ll 1$ and $e_{ij}$ are the components of the strain tensor given by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

The diffusion equation has the form Sherief et al. (2004)

$$D\beta_2 e_{kk,ii} + DaT_{,ii} + \dot{C} + \tau\ddot{C} - DbC_{,ii} = 0, \tag{4}$$

where $D$ is the diffusion coefficient, $b$ is a measure of diffusive effect and $\tau$ is the diffusion relaxation time.

The constitutive equations have the form Sherief et al. (2004)

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(T - T_0) - \beta_2 C], \tag{5a}$$

$$P = -\beta_1 e_{kk} + bC - a(T - T_0), \tag{5b}$$

where $\sigma_{ij}$ are the components of the stress tensor and $P$ is the chemical potential.

It follows from the description of the problem that all considered functions will depend on $x$ and $t$ only. We thus obtain the displacement components of the form

$$u_x = u(x,t), \quad u_y = u_z = 0. \tag{6}$$

The strain components are given by

$$e_{xx} = \mathscr{D}u, \quad e_{yy} = e_{zz} = e_{xy} = e_{yz} = e_{zx} = 0,$$

where $\mathscr{D} = \frac{\partial}{\partial x}$.

The cubical dilatation $e = e_{kk}$ is equal to

$$e = \mathscr{D}u. \tag{7}$$

From Eq. (5a), it follows that the stress tensor components have the form

$$\sigma = \sigma_{xx} = (\lambda + 2\mu)\mathscr{D}u - \beta_1(T - T_0) - \beta_2 C, \tag{8a}$$

$$\sigma_{yy} = \sigma_{zz} = \lambda\mathscr{D}u - \beta_1(T - T_0) - \beta_2 C, \tag{8b}$$

$$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0. \tag{9}$$

Eqs. (1), (2) and (4) thus reduce to

$$\rho\ddot{u} = \mu\mathscr{D}^2 u + (\lambda + \mu)\mathscr{D}e - \beta_1\mathscr{D}T - \beta_2\mathscr{D}C, \tag{10}$$

$$k\mathscr{D}^2 T = \rho c_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0(\mathscr{D}\dot{u} + \tau_0\mathscr{D}\ddot{u}) + aT_0(\dot{C} + \tau_0\ddot{C}), \tag{11}$$

$$D\beta_2\mathscr{D}^2 e + Da\mathscr{D}^2 T + \dot{C} + \tau\ddot{C} - Db\mathscr{D}^2 C = 0. \tag{12}$$

By using Eq. (7), Eqs. (10)–(12) can be written as

$$\rho\ddot{u} = (\lambda + 2\mu)\mathscr{D}e - \beta_1\mathscr{D}T - \beta_2\mathscr{D}C, \tag{13}$$

$$k\mathscr{D}^2 T = \rho c_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0(\dot{e} + \tau_0\ddot{e}) + aT_0(\dot{C} + \tau_0\ddot{C}), \tag{14}$$

$$D\beta_2\mathscr{D}^2 e + Da\mathscr{D}^2 T + \dot{C} + \tau\ddot{C} - Db\mathscr{D}^2 C = 0. \tag{15}$$

The governing equations can be put in a more convenient form by using the following non-dimensional variables

$$x^* = c_1\eta x, \quad u^* = c_1\eta u, \quad t^* = c_1^2\eta t, \quad \tau_0^* = c_1^2\eta\tau_0, \quad \tau^* = c_1^2\eta\tau,$$

$$\theta^* = \frac{\beta_1(T - T_0)}{\lambda + 2\mu}, \quad C^* = \frac{\beta_2 C}{\lambda + 2\mu}, \quad P^* = \frac{P}{\beta_2}, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\lambda + 2\mu},$$

where $c_1^2 = (\lambda + 2\mu)/\rho$, $\eta = \rho c_E/k$.

Using the above non-dimensional variables Eqs. (13)–(15), take the following form where we have dropped the asterisks for convenience

$$\ddot{u} = \mathscr{D}^2 u - \mathscr{D}\theta - \mathscr{D}C, \tag{16}$$

$$\mathscr{D}^2\theta = \dot{\theta} + \tau_0\ddot{\theta} + \varepsilon\dot{e} + \tau_0\ddot{e} + \varepsilon\alpha_1(\dot{C} + \tau_0\ddot{C}), \tag{17}$$

$$\mathscr{D}^2 e + \alpha_1\mathscr{D}^2\theta + \alpha_2(\dot{C} + \tau\ddot{C}) - \alpha_3\mathscr{D}^2 C = 0, \tag{18}$$

where

$$\varepsilon = \frac{\beta_1^2 T_0}{\rho c_E(\lambda + 2\mu)}, \quad \alpha_1 = \frac{a(\lambda + 2\mu)}{\beta_1\beta_2}, \quad \alpha_2 = \frac{\lambda + 2\mu}{\beta^2 D\eta}, \quad \alpha_3 = \frac{b(\lambda + 2\mu)}{\beta_2^2}.$$

Also Eqs. (5b) and (8) take the form

$$\sigma_{xx} = e - \theta - C, \tag{19a}$$

$$\sigma_{yy} = \sigma_{zz} = (1 - 2/\beta^2)e - \theta - C, \tag{19b}$$

$$P = \alpha_3 C - e - \alpha_1\theta, \tag{20}$$

where $\beta^2 = (\lambda + 2\mu)/\mu$.

The initial conditions of the problem are taken to be homogeneous while the boundary conditions are assumed to be

$$\sigma(x,t)|_{x=0} = 0, \quad \sigma(x,t)|_{x=\infty} = 0, \tag{21}$$

$$\theta(x,t)|_{x=0} = f_1(t), \quad \theta(x,t)|_{x=\infty} = 0, \tag{22}$$

$$P(x,t)|_{x=0} = f_2(t), \quad P(x,t)|_{x=\infty} = 0, \tag{23}$$

where $f_1(t)$ and $f_2(t)$ are known functions of $t$.

## 3. Solution in the Laplace transform domain

Introducing the Laplace transform defined by the formula

$$\bar{f}(s) = \int_0^\infty \mathrm{e}^{-st} f(t)\,\mathrm{d}t,$$

into Eqs. (16–19) and (20) and using the homogeneous initial conditions, we obtain

$$s^2\bar{u} = \mathscr{D}^2\bar{u} - \mathscr{D}\bar{\theta} - \mathscr{D}\overline{C}, \tag{24}$$

$$\mathscr{D}^2\bar{\theta} = (s + \tau_0 s^2)[\bar{\theta} + \varepsilon\bar{e} + \varepsilon\alpha_1\overline{C}], \tag{25}$$

$$\mathscr{D}^2\bar{e} + \alpha_1\mathscr{D}^2\bar{\theta} + [\alpha_2(s + \tau s^2) - \alpha_3\mathscr{D}^2]\overline{C} = 0, \tag{26}$$

$$\bar{\sigma}_{xx} = \bar{e} - \bar{\theta} - \overline{C}, \tag{27a}$$

$$\bar{\sigma}_{yy} = \bar{\sigma}_{zz} = (1 - 2/\beta^2)\bar{e} - \bar{\theta} - \overline{C}, \tag{27b}$$

$$\overline{P} = \alpha_3\overline{C} - \bar{e} - \alpha_1\bar{\theta}. \tag{28}$$

Taking the divergence of Eq. (24), we obtain

$$(\mathscr{D}^2 - s^2)\bar{e} - \mathscr{D}^2\bar{\theta} - \mathscr{D}^2\overline{C} = 0. \tag{29}$$

Eliminating $\bar{e}$ and $\overline{C}$ between Eqs. (25), (26) and (29), we obtain

$$(\mathscr{D}^6 - a_1\mathscr{D}^4 + a_2\mathscr{D}^2 - a_3)\bar{\theta} = 0, \tag{30}$$

where

$$a_1 = \frac{s}{\alpha_3 - 1}[(1 + \tau_0 s)(\alpha_1\varepsilon(\alpha_1 + 2) + \alpha_3(\varepsilon + 1) - 1) + \alpha_2(1 + \tau s) + \alpha_3 s],$$

$$a_2 = \frac{s^2}{\alpha_3 - 1}[(1 + \tau_0 s)(\varepsilon s\alpha_1^2 + \alpha_3 s + \alpha_2(\varepsilon + 1)(1 + \tau s)) + \alpha_2 s(1 + \tau s)],$$

$$a_3 = \frac{s^4\alpha_2}{\alpha_3 - 1}(1 + \tau s)(1 + \tau_0 s).$$

In a similar manner we can show that $\bar{e}$ and $\overline{C}$ satisfy the equations

$$(\mathscr{D}^6 - a_1\mathscr{D}^4 + a_2\mathscr{D}^2 - a_3)\bar{e} = 0, \tag{31}$$

$$(\mathscr{D}^6 - a_1\mathscr{D}^4 + a_2\mathscr{D}^2 - a_3)\overline{C} = 0, \tag{32}$$

Eq. (30) can be factorized as

$$(\mathscr{D}^2 - k_1^2)(\mathscr{D}^2 - k_2^2)(\mathscr{D}^2 - k_3^2)\bar{\theta} = 0, \tag{33}$$

where $k_1$, $k_2$ and $k_3$ are the roots with positive real parts of the characteristic equation

$$k^6 - a_1 k^4 + a_2 k^2 - a_3 = 0. \tag{34}$$

The solution of Eq. (33) bounded for $x \geqslant 0$ has the form

$$\bar{\theta}(x, s) = \sum_{i=1}^{3} A_i \mathrm{e}^{-k_i x}, \tag{35}$$

where $A_i = A_i(s)$ are parameters depending on s only. Note that the roots with negative real parts are not included in Eq. (35) since they are unbounded as $x \to \infty$. It is worth mentioning here that the roots $k_1$, $k_2$ and $k_3$ are functions of $s$.

Similarly, the solution of Eqs. (31) and (32) can be written as

$$\bar{e}(x, s) = \sum_{i=1}^{3} A_i' \mathrm{e}^{-k_i x}, \tag{36}$$

$$\overline{C}(x, s) = \sum_{i=1}^{3} A_i'' \mathrm{e}^{-k_i x}, \tag{37}$$

where $A_i'$ and $A_i''$ are parameters depending only on $s$.

Substituting from Eqs. (35)–(37) into Eqs. (25), (26) and (29), we get

$$A_i' = \frac{k_i^2[k_i^2 - (1 - \varepsilon\alpha_1)(s + \tau_0 s^2)]}{\varepsilon(s + \tau_0 s^2)[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]}A_i, \tag{38}$$

$$A_i'' = \frac{k_i^4 - k_i^2[s^2 + (\varepsilon + 1)(s + \tau_0 s^2)] + s^3(1 + \tau_0 s)}{\varepsilon(s + \tau_0 s^2)[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i. \tag{39}$$

We thus have

$$\bar{e}(x,s) = \sum_{i=1}^{3} \frac{k_i^2[k_i^2 - (1 - \varepsilon\alpha_1)(s + \tau_0 s^2)]}{\varepsilon(s + \tau_0 s^2)[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i \mathrm{e}^{-k_i x}, \tag{40}$$

$$\overline{C}(x,s) = \sum_{i=1}^{3} \frac{k_i^4 - k_i^2[s^2 + (\varepsilon + 1)(s + \tau_0 s^2)] + s^3(1 + \tau_0 s)}{\varepsilon(s + \tau_0 s^2)[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i \mathrm{e}^{-k_i x}. \tag{41}$$

Integrating both sides of Eq. (7) from $x$ to infinity, and assuming that $u$ vanishes at infinity, we obtain upon using the relation (40)

$$\bar{u}(x,s) = -\sum_{i=1}^{3} \frac{k_i[k_i^2 - (1 - \varepsilon\alpha_1)(s + \tau_0 s^2)]}{\varepsilon(s + \tau_0 s^2)[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i \mathrm{e}^{-k_i x}. \tag{42}$$

Substituting from Eqs. (35), (40) and (41) into Eqs. (27a) and (28), we get

$$\bar{\sigma}_{xx}(x,s) = \frac{s}{\varepsilon(1 + \tau_0 s)} \sum_{i=1}^{3} \frac{[k_i^2 - (1 - \varepsilon\alpha_1)(s + \tau_0 s^2)]}{[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i \mathrm{e}^{-k_i x}, \tag{43}$$

$$\overline{P}(x,s) = \frac{\alpha_2(1 + \tau s)}{\varepsilon(1 + \tau_0 s)} \sum_{i=1}^{3} \frac{k_i^4 - k_i^2[s^2 + (\varepsilon + 1)(s + \tau_0 s^2)] + s^3(1 + \tau_0 s)}{k_i^2[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i \mathrm{e}^{-k_i x}. \tag{44}$$

In order to evaluate the unknown parameters $A_1$, $A_2$ and $A_3$, we shall use the Laplace transform of the boundary conditions (21)–(23) together with Eqs. (35), (43) and (44). We thus arrive at the following set of linear equations

$$\sum_{i=1}^{3} \frac{[k_i^2 - (1 - \varepsilon\alpha_1)(s + \tau_0 s^2)]}{[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} A_i = 0, \tag{45}$$

$$\sum_{i=1}^{3} A_i = \bar{f}_1(s), \tag{46}$$

$$\sum_{i=1}^{3} \frac{k_i^4 - k_i^2[s^2 + (\varepsilon + 1)(s + \tau_0 s^2)] + s^3(1 + \tau_0 s)}{k_i^2[(1 + \alpha_1)k_i^2 - \alpha_1 s^2]} \quad A_i = \frac{\bar{f}_2(s)\varepsilon(1 + \tau_0 s)}{\alpha_2(1 + \tau s)}. \tag{47}$$

Solving the linear system of Eqs. (45)–(47), we can obtain the parameters $A_1 - A_3$. This completes the solution of the problem in the Laplace transform domain.

## 4. Inversion of the Laplace transform

We shall now outline briefly the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as Churchill (1972)

$$f(t) = \frac{1}{2\pi i}\int_{d-\mathrm{i}\infty}^{d+\mathrm{i}\infty} \mathrm{e}^{st}\bar{f}(s)\,\mathrm{d}s,$$

where $d$ is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$.

Taking $s = d + \mathrm{i}y$, and using Fourier series in the interval $[0, 2L]$, we obtain the approximate formula Honig and Hirdes (1984)

$$f(t) \cong f_N(t) = 1/2c_0 + \sum_{k=1}^{N} c_k, \quad \text{for } 0 \leqslant t \leqslant 2L,$$

where

$$c_k = \frac{\mathrm{e}^{\mathrm{d}t}}{L}\mathrm{Re}[\mathrm{e}^{\mathrm{i}k\pi t/L}\bar{f}(d + \mathrm{i}k\pi/L)]. \tag{48}$$

Two methods are used to reduce the total error. First, the 'Korrecktur' method is used to reduce the discretization error. Next, the $\varepsilon$ algorithm is used to reduce the truncation error and therefore to accelerate convergence. The details of these methods can be found in Honig and Hirdes (1984). The values of $d$ and $L$ are chosen according to the criteria outlined in Honig and Hirdes (1984).

## 5. Numerical results

For the purpose of numerical illustration, the problem was solved for the following choice of the functions $f_1(t)$ and $f_2(t)$

$$f_1(t) = \theta_0 H(t),$$

$$f_2(t) = P_0 H(t),$$

where $\theta_0$ and $P_0$ are constants and $H(t)$ is the Heaviside unit step function.

We thus have

$$\bar{f}_1(s) = \frac{\theta_0}{s},$$

$$\bar{f}_2(s) = \frac{P_0}{s}.$$

The roots $k_1$, $k_2$ and $k_3$ of the characteristic equation are given by

$$k_1 = \sqrt{\frac{1}{3}[2p\sin(q) + a_1]},$$

$$k_2 = \sqrt{\frac{1}{3}\left[a_1 - p(\sqrt{3}\cos(q) + \sin(q))\right]},$$

$$k_3 = \sqrt{\frac{1}{3}\left[a_1 + p(\sqrt{3}\cos(q) - \sin(q))\right]},$$

where

$$p = \sqrt{a_1^2 - 3a_2}, \quad q = \frac{\sin^{-1}(r)}{3} \quad \text{and } r = -\frac{2a_1^3 - 9a_1a_2 + 27a_3}{2p^3}.$$

The copper material was chosen for purposes of numerical evaluations. The material constants of the problem are thus given by in SI units Thomas (1980)

$$T_0 = 293\ \text{K},\quad \sigma = 8954\ \text{kg/m}^3,\quad \tau_0 = 0.02\ \text{s},\quad \tau = 0.2\ \text{s},$$

$$c_E = 383.1\ \text{J/(kg K)},\quad \alpha_t = 1.78(10)^{-5}\ \text{K}^{-1},\quad k = 386\ \text{W/(m K)},$$

$$\lambda = 7.76(10)^{10}\ \text{kg/(m s}^2),\quad \mu = 3.86(10)^{10}\ \text{kg/(m s}^2),$$

$$\alpha_c = 1.98(10)^{-4}\ \text{m}^3/\text{kg};\quad D = 0.85(10)^{-8}\ \text{kg s/m}^3;\quad a = 1.2(10)^4\ \text{m}^2/(\text{s}^2\ \text{K});\quad b = .9(10)^6\ \text{m}^5/(\text{kg s}^2).$$

Using these values, it was found that $\eta = 8886.73$, $\varepsilon = 0.0168$, $\beta^2 = 4$, $\alpha_1 = 5.43$, $\alpha_2 = 0.533$ and $\alpha_3 = 36.24$.

It should be noted that a unit of non-dimensional time corresponds to $6.5(10)^{-12}$ s while a unit of non-dimensional length corresponds to $2.7(10)^{-8}$ m.

The computations were carried out for two values of non-dimensional time, namely for $t = 0.05$ and $t = 0.075$. The temperature, displacement, stress, concentration and chemical potential are shown in Figs. 1–5, respectively. Dashed lines represent the case when $t = 0.075$, while solid lines represent the case when $t = 0.05$.

In all these figures, it is clear that all the functions considered have a non-zero value only in a bounded region of space and vanish identically outside this region. This region expands with the passage of time. The

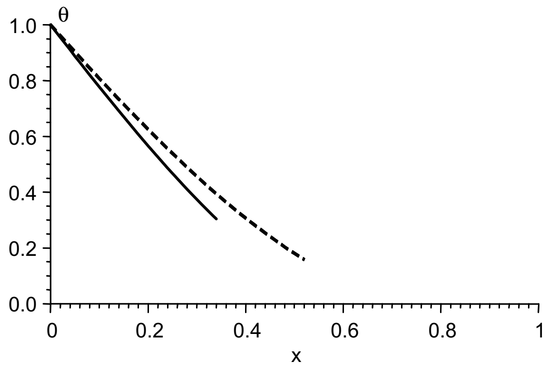


Fig. 1. Temperature distribution.

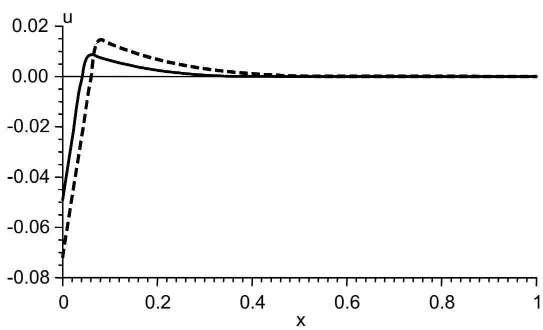


Fig. 2. Displacement distribution.

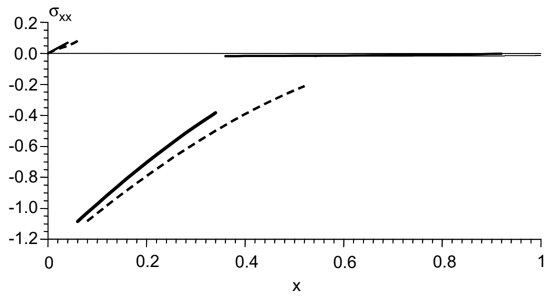


Fig. 3. Stress distribution.

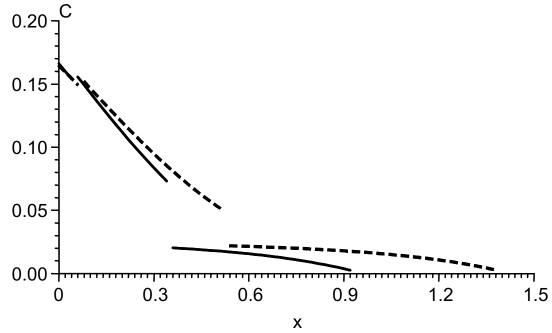


Fig. 4. Concentration distribution.

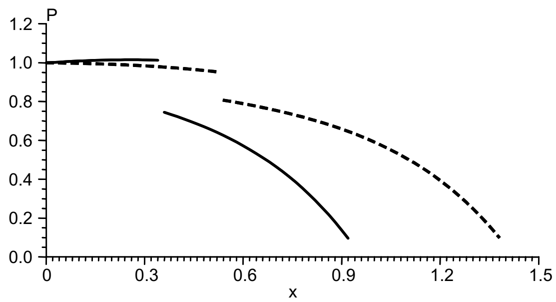


Fig. 5. Chemical potential distribution.

edge of this region is the wave front which moves with a finite speed. This is not the case for the coupled and uncoupled theories of thermoelasticity where an infinite speed of propagation is inherent and hence all the considered functions have a non-zero (although may be very small) values for any point in the medium. Although these results were calculated numerically with a precision of five digits, the solution for $x > 0.923$ when $t = 0.05$ and for $x = 1.38$ was found to be zero correct to 12 decimal digits indicating that the solution in this region is identically equal to zero.

For $t = 0.05$, we have three wave fronts at the positions $x = 0.047$, $x = 0.352$ and $x = 0.923$, approximately. Due to the coupling between the governing equations, the arrival of any wave front at a certain position affects all the considered functions. By numerical experimentation on the values of the functions just before and just after the arrival of the wave fronts and by analogy to the wave propagation in generalized thermoelasticity Sherief and Hamza (1994), it was found that the first and second waves are mainly thermo-mechanical in nature while the third wave affects diffusion mainly. The topic of wave propagation in the theory of generalized thermoelastic diffusion is being treated by the authors and will be reported in a future paper.

We note that the three wave fronts at the positions indicated above do not show for the temperature (Fig. 1), the displacement (Fig. 2), the stress (Fig. 3), and the chemical potential (Fig. 5) because the values of these discontinuities are very small (of the order of 0.001).

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